Remainder Theorem

Let $f(x) = 2x^3 + ax^2 + bx + 1$. If $f(x)$ is divided by $(x+1)$, the remainder is 5 and when it is divided by $(x-4)$, the remainder is also 5.

PAGE 6 Determine the values of a and b.

Solution:

$x + 1 = 0$

$x = -1$

$f(-1) = 5$

$f(-1) = 2(-1)^3 + a(-1)^2 + b(-1) + 1$

$f(-1) = -2 + a - b + 1$

$f(-1) = a - b - 1$

$a - b - 1 = 5$

$a - b = 6$

$a = b + 6$ ..... eq ①

$x - 4 = 0$

$x = 4$

$f(4) = 5$

$f(4) = 2(4)^3 + a(4)^2 + b(4) + 1$

$f(4) = 128 + 16a + 4b + 1$

$f(4) = 16a + 4b + 129$

$16a + 4b + 129 = 5$

$16a + 4b = -124$ ..... eq. ②

Substitute eq. ① into eq. ②

$16(b+6) + 4b = -124$

$16b + 96 + 4b = -124$

$\frac{20b}{20} = \frac{-220}{20}$

$b = -11$

Substitute b into eq. ①

$a = -11 + 6$

$a = -5$