

Remainder Theorem

Typical Exam questions

Example 1:

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Let $f(x) = 2x^3 + ax^2 + bx + 1$. If $f(x)$ is divided by $(x+1)$, the remainder is 5 and when it is divided by $(x-4)$, the remainder is also 5. Determine the values of a and b .

Solution

$f(x) = 2x^3 + ax^2 + bx + 1$ is divided by $(x+1)$ means

$$\begin{aligned} \cancel{f(x)} & \quad 1) \quad x+1=0 \\ & \quad \quad x=-1 \end{aligned}$$

$$2) \quad f(-1) = 2(-1)^3 + a(-1)^2 + b(-1) + 1$$

$$f(-1) = a - b - 1$$

\therefore The remainder is 5 means $f(-1) = 5$

$$\therefore f(-1) = a - b - 1$$

$$\text{and } f(-1) = 5$$

$$\therefore a - b - 1 = 5$$

$$a - b = 6$$

$$\therefore a = b + 6 \quad \dots \dots \textcircled{1}$$

When $f(x)$ is divided by $(x-4)$, it means

$$x-4=0$$

$$x=4$$

$$\therefore f(4) = 2(4)^3 + a(4)^2 + b(4) + 1$$

$$f(4) = 16a + 4b + \del{129} 129$$

$$\text{Also } f(4) = 5$$

$$\therefore 16a + 4b + 129 = 5$$

$$16a + 4b = -124 \quad \dots \dots \textcircled{2}$$

Substitute eq ① into eq ②

$$\therefore 16(b+6) + 4b = -124$$

$$16b + 96 + 4b = -124$$

$$\frac{20b}{20} = \frac{-220}{20}$$

Substitute b into ①

$$\hookrightarrow a = -11 + 6$$

$$\underline{a = -5}$$