Given that $x=1$ is a root of the cubic equation $x^3-5x^2+17x-13=0$. Determine the other two roots, expressing any non-real solutions in terms of i

It means when $x^3-5x^2+17x-13$ is divided by $x-1$, the remainder is 0.

∴ This also means means that $x-1$ is a factor of $x^3-5x^2+17x-13$

Solution

Since we know that $x-1$ is a factor of $x^3-5x^2+17x-13$, we divide $x^3-5x^2+17x-13$ by $x-1$

$$
\begin{array}{r|l}
x-1 & x^2-4x+13 \\
\hline
 & x^3-5x^2+17x-13 \\
 & -x^3+x^2 \\
\hline
 & \quad -4x^2+17x \\
 & \quad +4x^2-4x \\
\hline
 & \qquad 13x-13 \\
 & \qquad -13x+13 \\
\hline
 & \qquad 0
\end{array}
$$

$\therefore x^3-5x^2+17x-13=0$

$\therefore (x-1)(x^2-4x+13)=0$

$x-1=0$ or $x^2-4x+13=0$

Use quadratic formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(13)}}{2(1)}$$

$$x=\frac{4\pm 6i}{2}$$

$$x=\frac{4+6i}{2} \quad \text{OR} \quad x=\frac{4-6i}{2}$$

$x=1$ OR $x=2+3i$ OR $x=2-3i$