THE REMAINDER THEOREM

- We have used long division to find the remainder when dividing two polynomials, however long-division is a time-consuming method.
- Remainder theorem can be used to:
  * factorise third-degree polynomials and
  * find the remainder and quotient when we divide a third-degree polynomial by a linear polynomial.

Remainder Theorem states that:

When a third-degree polynomial $f(x)$ is divided by a linear polynomial $x-a$, the remainder of that division will be equivalent to $f(a)$.

Example:

$5x^3+3x^2-2x+3 \div 2x-3$

Using LONG DIVISION

$$\frac{5}{2}x^2 + \cancel{2x}\ \frac{21}{4}x + \frac{55}{8}$$

$$2x-3 \,\big)\, 5x^3+3x^2-2x+3$$

$$-\left(5x^3 - \frac{15}{2}x^2\right)$$

$$\frac{21}{2}x^2 - 2x$$

$$-\left(\frac{21}{2}x^2 - \frac{63}{4}x\right)$$

$$\frac{55}{4}x + 3$$

$$-\left(\frac{55}{4}x - \frac{165}{8}\right)$$

$$\frac{189}{8}$$

$\therefore$ Remainder $= \frac{189}{8}$

Quotient $= \frac{5}{2}x^2 + \frac{21}{4}x + \frac{55}{8}$

Using REMAINDER THEOREM

① Equate $2x-3$ to 0

$\therefore 2x-3=0$

② $\therefore x=\frac{3}{2}$

Let $f(x)=5x^3+3x^2-2x+3$

$\therefore f\left(\frac{3}{2}\right) = 5\left(\frac{3}{2}\right)^3 + 3\left(\frac{3}{2}\right)^2 - 2\left(\frac{3}{2}\right)+3$

$\therefore f\left(\frac{3}{2}\right) = 5\left(\frac{27}{8}\right) + 3\left(\frac{9}{4}\right) - 3 + 3$

$\therefore f\left(\frac{3}{2}\right) = \frac{189}{8}$ ← Remainder